Matrice

1. $A = \begin{pmatrix} -1 & 2 \\ 0 & 3 \\ -4 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 0 & -2 \\ 1 & 5 \\ -3 & 4 \end{pmatrix}$. Naći matricu $4A - 3B$

$$4A - 3B = \begin{pmatrix} -4 & 8 \\ 0 & 12 \\ -16 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -6 \\ 3 & 15 \\ -9 & 12 \end{pmatrix} = \begin{pmatrix} -4 & 14 \\ -3 & -3 \\ -7 & 0 \end{pmatrix}$$

2. Riješiti matričnu j-nu $-2(X+A) = 3X + B$ gdje je

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \quad i \quad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$-2(X+A) = 3X + B$$

$$-2X - 2A = 3X + B$$

$$5X = -2A - B$$

$$X = \frac{1}{5}(-2A - B)$$

$$-2A - B = \begin{pmatrix} -2 & -4 & -6 \\ 0 & -2 & -4 \\ 0 & 0 & -2 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -3 & -4 & -6 \\ 0 & -2 & -4 \\ 0 & 0 & -3 \end{pmatrix}$$

$$X = \frac{1}{5}\begin{pmatrix} -3 & -4 & -6 \\ 0 & -2 & -4 \\ 0 & 0 & 3 \end{pmatrix} =$$

3. Neka je $A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{pmatrix}$ i $B = \begin{pmatrix} 2 & 0 & -4 \\ 3 & 1 & 5 \end{pmatrix}$. Naći matricu $C = 3A \cdot B^T - BA^T$

$$C_{m\times p} = A_{m\times n} \cdot B_{n\times p}$$

$$A \cdot B^T = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{pmatrix}_{2\times3} \cdot \begin{pmatrix} 2 & 3 \\ 0 & 1 \\ -4 & 5 \end{pmatrix}_{3\times2} = \begin{pmatrix} 1\cdot2+(-1)\cdot0+2\cdot(-4) & 1\cdot3+(-1)\cdot1+2\cdot5 \\ 0\cdot2+3\cdot0+4\cdot(-4) & 0\cdot3+3\cdot1+4\cdot5 \end{pmatrix} = \begin{pmatrix} -6 & 12 \\ -16 & 23 \end{pmatrix}$$

$$B \cdot A^T = \begin{pmatrix} 2 & 0 & -4 \\ 3 & 1 & 5 \end{pmatrix}_{2\times3} \cdot \begin{pmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 4 \end{pmatrix}_{3\times2} = \begin{pmatrix} 2\cdot1+0\cdot(-1)+(-4)\cdot2 & 2\cdot0+0\cdot3+(-4)\cdot4 \\ 3\cdot1+1\cdot(-1)+5\cdot2 & 3\cdot0+1\cdot3+5\cdot4 \end{pmatrix} = \begin{pmatrix} -6 & -16 \\ 12 & 23 \end{pmatrix}$$

$$C = 3\begin{pmatrix} -6 & 12 \\ -16 & 23 \end{pmatrix} - \begin{pmatrix} -6 & -16 \\ 12 & 23 \end{pmatrix} = \begin{pmatrix} -18 & 36 \\ -48 & 69 \end{pmatrix} - \begin{pmatrix} -6 & -16 \\ 12 & 23 \end{pmatrix} = \begin{pmatrix} -12 & 52 \\ -60 & 46 \end{pmatrix}$$

$$(A^T)^T = A$$

$$(kA)^T = k \cdot A^T, \ k \in \mathbb{R}$$

$$(A+B)^T = A^T + B^T$$

$$(\underbrace{A \cdot B}\cdot C)^T = C^T (A \cdot B)^T = C^T B^T A^T$$

$$(A \cdot B)^T = B^T \cdot A^T$$

$A\cdot B - 3A = A(B-3E)$!!!

Determinante

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \sum_{\pi \in S_3} (-1)^{N(\pi)} \cdot a_{1\pi(1)} \cdot a_{2\pi(2)} \cdot a_{3\pi(3)}$$

$$\begin{array}{cccc} \pi: & 1 & 2 & 3 \\ & 2 & 3 & 1 \\ & \downarrow & \downarrow & \downarrow \\ & \pi(1) & \pi(2) & \pi(3) \end{array}$$

$|S_3| = 3! = 6$ broj permutacija za skup $\{1,2,3\}$

S_3 - skup svih permutacija skupa $\{1,2,3\}$

$\pi_1 : 1\ 2\ 3 \quad N(\pi_1)=0$

$\pi_2 : 1\ 3\ 2 \quad N(\pi_2)=1$

$\pi_3 : 2\ 1\ 3 \quad N(\pi_3)=1$

$\pi_4 : 2\ 3\ 1 \quad N(\pi_4)=2$

$\pi_5 : 3\ 1\ 2 \quad N(\pi_5)=2$

$\pi_6 : 3\ 2\ 1 \quad N(\pi_6)=3$

1. Koji znak u determinanti 6-tog reda ima proizvod $a_{32}\cdot a_{43}\cdot a_{14}\cdot a_{51}\cdot a_{66}\cdot a_{25}$

$a_{14}\cdot a_{25}\cdot a_{32}\cdot a_{43}\cdot a_{51}\cdot a_{66}$

Ovaj proizvod je formiran pomoću permutacije $\pi: 4\ 5\ 2\ 3\ 1\ 6$

$N(\pi) = 3+3+1+1 = 8$ inverzija

$(-1)^{N(\pi)} = (-1)^8 = 1$

Proizvod ima predznak $+$.

2. $\begin{vmatrix} 2 & -4 \\ 5 & 6 \end{vmatrix} = 2\cdot 6 - (-4)\cdot 5 = 12+20 = 32$

$$\left|\begin{matrix} 2 & 0 & 3 \\ 1 & -1 & 4 \\ -2 & 0 & 3 \end{matrix}\right| \begin{matrix} 2 & 0 \\ 1 & -1 \\ -2 & 0 \end{matrix} = -6+0+0-6-0-0 = -12$$

Sarusovo pravilo

$P_1 + P_2 + P_3 - (P_1' + P_2' + P_3')$

$$\begin{vmatrix} 2 & 0 & 3 \\ 1 & -1 & 4 \\ -2 & 0 & 3 \end{vmatrix} = -6+0+0-(6+0+0) = -12$$

$$\to \begin{vmatrix} 2 & 0 & 3 \\ 1 & -1 & 4 \\ -2 & 0 & 3 \end{vmatrix} = 1\cdot(-1)^{2+1}\cdot\begin{vmatrix} 0 & 3 \\ 0 & 3 \end{vmatrix} + (-1)\cdot(-1)^{2+2}\begin{vmatrix} 2 & 3 \\ -2 & 3 \end{vmatrix} + 4\cdot(-1)^{2+3}\begin{vmatrix} 2 & 0 \\ -2 & 0 \end{vmatrix} =$$

$$= -0 - 12 - 4\cdot 0 = -12$$

Laplasov razvoj

24. oktobar

1. Izračunati determinantu

$$D = \begin{vmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 1 \\ 0 & 0 & 0 & \dots & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 & 0 \end{vmatrix}_n = (-1)^{N(\pi)} a_{1n} a_{2n-1} \cdot \ldots \cdot a_{n-12} a_{n1} =$$

$$= (-1)^{\frac{(n-1)n}{2}} \cdot 1 \cdot 1 \cdot \ldots \cdot 1 = (-1)^{\frac{(n-1)n}{2}}$$

$$1 + \ldots + k = \frac{k(k+1)}{2}$$

$\pi : n \quad n-1 \quad \ldots \quad 2 \quad 1$

$$N(\pi) = n-1 + n-2 + \ldots + 1 = \frac{(n-1)n}{2}$$

2.
$$\begin{vmatrix} a_{11} & \sim & \\ & a_{22} & \sim \\ 0 & & \ddots & \sim \end{vmatrix} = a_{11} a_{22} a_{33} \ldots a_{nn}$$

2. Koristeći osobine determinanti dokazati identitete

a) $$\begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ c+a & b & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ c+a & b & 1 \end{vmatrix} = \begin{vmatrix} a+b+c & c & 1 \\ a+b+c & a & 1 \\ a+b+c & b & 1 \end{vmatrix} = (a+b+c) \begin{vmatrix} 1 & c & 1 \\ 1 & a & 1 \\ 1 & b & 1 \end{vmatrix} = 0$$

b) $$\begin{vmatrix} b & a & a \\ a & b & a \\ a & a & b \end{vmatrix} = (2a+b)(b-a)^2$$

$$\begin{vmatrix} b & a & a \\ a & b & a \\ a & a & b \end{vmatrix} = \begin{vmatrix} 2a+b & a & a \\ 2a+b & b & a \\ 2a+b & a & b \end{vmatrix} = (2a+b) \begin{vmatrix} 1 & a & a \\ 1 & b & a \\ 1 & a & b \end{vmatrix} = (2a+b) \begin{vmatrix} 1 & a & a \\ 0 & b-a & 0 \\ 0 & 0 & b-a \end{vmatrix} =$$

$$= (2a+b)(b-a)^2$$

c) $$\begin{vmatrix} ax & a^2+x^2 & 1 \\ ay & a^2+y^2 & 1 \\ az & a^2+z^2 & 1 \end{vmatrix} = a(x-y)(x-z)(z-y)$$

$$\begin{vmatrix} ax & a^2+x^2 & 1 \\ ay & a^2+y^2 & 1 \\ az & a^2+z^2 & 1 \end{vmatrix} = a \begin{vmatrix} x & a^2+x^2 & 1 \\ y & a^2+y^2 & 1 \\ z & a^2+z^2 & 1 \end{vmatrix} = a \left(\begin{vmatrix} x & a^2 & 1 \\ y & a^2 & 1 \\ z & a^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} \right) =$$

$$= a \left(\underbrace{a^2 \begin{vmatrix} x & 1 & 1 \\ y & 1 & 1 \\ z & 1 & 1 \end{vmatrix}}_{0} - \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix} \right) = a \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = a \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} =$$

$$= a(y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix} = a(y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 0 & z-y \end{vmatrix} = a(y-x)(z-x)(z-y)$$

$$= a(x-y)(x-z)(z-y)$$

3. Izračunati determinantu n-tog reda

$$D = \begin{vmatrix} a & x & x & \dots & x \\ x & a & x & \dots & x \\ x & x & a & \dots & x \\ \vdots & \vdots & \vdots & & \vdots \\ x & x & x & \dots & a \end{vmatrix} = \begin{vmatrix} a+(n-1)x & a+(n-1)x & a+(n-1)x & \dots & a+(n-1)x \\ x & a & x & \dots & x \\ x & x & a & \dots & x \\ \vdots & \vdots & \vdots & & \vdots \\ x & x & x & \dots & a \end{vmatrix} =$$

$$= (a+(n-1)x) \begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ x & a & x & \dots & x \\ x & x & a & \dots & x \\ \vdots & \vdots & \vdots & & \vdots \\ x & x & x & \dots & a \end{vmatrix} = (a+(n-1)x) \begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & a-x & 0 & \dots & 0 \\ 0 & 0 & a-x & \dots & 0 \\ \vdots & \vdots & & & \vdots \\ 0 & 0 & 0 & \dots & a-x \end{vmatrix} =$$

$$= (a+(n-1)x)(a-x)^{n-1}$$

4. Izračunati determinantu $D = (a_{ij})$ ako je $a_{ij} = \min(i,j)$

$$\cdot(-3) \quad \cdot(-2) \quad \cdot(-n) \quad \cdot(-(n-1))$$

$$D = \begin{vmatrix} 1 & 1 & 1 & \dots & 1 & 1 \\ 1 & 2 & 2 & \dots & 2 & 2 \\ 1 & 2 & 3 & \dots & 3 & 3 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 2 & 3 & \dots & n-1 & n-1 \\ 1 & 2 & 3 & \dots & n-1 & n \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & \dots & 1 & 1 \\ -1 & 0 & 0 & \dots & 0 & 0 \\ -2 & -1 & 0 & \dots & 0 & 0 \\ \vdots & & & & & \\ 1 & & & & & \end{vmatrix} = \begin{vmatrix} 1 & -1 & -2 & \dots & 2-n & 1-n \\ 1 & 0 & -1 & \dots & 3-n & 2-n \\ 1 & 0 & 0 & \dots & 4-n & 3-n \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 0 & 0 & \dots & 0 & -1 \\ 1 & 0 & 0 & \dots & 0 & 0 \end{vmatrix}$$

$$= 1 \cdot (-1)^{n+1} \begin{vmatrix} -1 & -2 & \dots & 2-n & 1-n \\ 0 & -1 & \dots & 3-n & 2-n \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 0 & -1 \end{vmatrix}_{n-1} = 1 \cdot (-1)^{n+1} \cdot (-1)^{n-1} = (-1)^{2n} = 1$$

Matrice

1. Riješiti matričnu j-nu, pa naći X ako je $A=\begin{pmatrix} 2 & 1 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{pmatrix}$, $B=\begin{pmatrix} -2 & -1 & 0 \\ 0 & -2 & -1 \\ 0 & 0 & -2 \end{pmatrix}$

$$A+B\cdot(X^T)^{-1}=E+A\cdot(X^T)^{-1}$$

$$B(X^T)^{-1}-A(X^T)^{-1}=E-A$$

$$(B-A)^{-1}\cdot / \; (B-A)(X^T)^{-1}=E-A$$

$$(X^T)^{-1}=\underbrace{(B-A)^{-1}}\,\underbrace{(E-A)} \; /^{-1}$$

$$X^T=(E-A)^{-1}\cdot(B-A) \; /^{T}$$

$$X=(B-A)^T\left((E-A)^{-1}\right)^T$$

$(A^{-1})^{-1}=A$

$(A+B)^{-1}\neq A^{-1}+B^{-1}$

$(A\cdot B)^{-1}=B^{-1}\cdot A^{-1}$

$A\cdot A^{-1}=E$

$A\cdot E=A$

$E^{-1}=E$

$$C=(B-A)^T$$

$$C=\begin{pmatrix} -4 & -2 & -3 \\ 0 & -5 & -2 \\ 0 & 0 & -6 \end{pmatrix}^T=\begin{pmatrix} -4 & 0 & 0 \\ -2 & -5 & 0 \\ -3 & -2 & -6 \end{pmatrix}$$

$$G=E-A=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}-\begin{pmatrix} 2 & 1 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{pmatrix}=\begin{pmatrix} -1 & -1 & -3 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{pmatrix}$$

$$\det G=\begin{vmatrix} -1 & -1 & -3 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{vmatrix}=-6\neq 0 \Rightarrow \text{postoji } G^{-1}$$

$G_{11}=(-1)^{1+1}\begin{vmatrix} -2 & -1 \\ 0 & -3 \end{vmatrix}=6$ $\quad G_{21}=(-1)^{2+1}\begin{vmatrix} -1 & -3 \\ 0 & -3 \end{vmatrix}=-3$ $\quad G_{31}=(-1)^{3+1}\begin{vmatrix} -1 & -3 \\ -2 & -1 \end{vmatrix}=-5$

$G_{12}=(-1)^{1+2}\begin{vmatrix} 0 & -1 \\ 0 & -3 \end{vmatrix}=0$ $\quad G_{22}=(-1)^{2+2}\begin{vmatrix} -1 & -3 \\ 0 & -3 \end{vmatrix}=3$ $\quad G_{32}=(-1)^{3+2}\begin{vmatrix} -1 & -3 \\ 0 & -1 \end{vmatrix}=-1$

$G_{13}=(-1)^{1+3}\begin{vmatrix} 0 & -2 \\ 0 & 0 \end{vmatrix}=0$ $\quad G_{23}=(-1)^{2+3}\begin{vmatrix} -1 & -1 \\ 0 & 0 \end{vmatrix}=0$ $\quad G_{33}=(-1)^{3+3}\begin{vmatrix} -1 & -1 \\ 0 & -2 \end{vmatrix}=2$

$$G^{-1}=\frac{1}{\det G}\begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}^T=\frac{1}{-6}\begin{pmatrix} 6 & -3 & -5 \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

$$X=C\cdot(G^{-1})^T$$

$$X = -\frac{1}{6}\begin{pmatrix} -4 & 0 & 0 \\ -2 & -5 & 0 \\ -3 & -2 & -6 \end{pmatrix}\begin{pmatrix} 6 & 0 & 0 \\ -3 & 3 & 0 \\ -5 & -1 & 2 \end{pmatrix} = -\frac{1}{6}\begin{pmatrix} -24 & 0 & 0 \\ 3 & -15 & 0 \\ 18 & 0 & -12 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ -\frac{1}{2} & \frac{5}{2} & 0 \\ -3 & 0 & 2 \end{pmatrix}$$

2. $(XA+B)^{-1} \cdot (XD+B) = D$ $\qquad A = \begin{pmatrix} -1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

$(XA+B)\cdot$ $\qquad D = \begin{pmatrix} -1 & 1 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$

$$XD + B = (XA+B)D$$

$$XD + B = XAD + BD$$

$$XD - XAD = BD - B$$

$$X(D - AD) = BD - B \quad / \cdot (D-AD)^{-1}$$

$$X = (\underbrace{BD}_{C} - B)(D - \underbrace{AD}_{G})^{-1}$$

$C = BD$

$$C = \begin{pmatrix} 2 & -1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad H = C - B = \begin{pmatrix} -4 & 4 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$G = AD$

$$G = \begin{pmatrix} -1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 1 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -3 & 8 \\ 0 & -2 & 2 \\ 0 & 0 & 6 \end{pmatrix} \qquad K = D - G = \begin{pmatrix} -2 & 4 & -6 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{pmatrix}$$

$$\det K = \begin{vmatrix} -2 & 4 & -6 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{vmatrix} = 8 \neq 0 \Rightarrow \text{postoji !}$$

$K_{11} = (-1)^{1+1}\begin{vmatrix} 1 & 0 \\ 0 & -4 \end{vmatrix} = -4 \qquad K_{21} = (-1)^{2+1}\begin{vmatrix} 4 & -6 \\ 0 & -4 \end{vmatrix} = 16 \qquad K_{31} = (-1)^{3+1}\begin{vmatrix} 4 & -6 \\ 1 & 0 \end{vmatrix} = 6$

$K_{12} = (-1)^{1+2}\begin{vmatrix} 0 & 0 \\ 0 & -4 \end{vmatrix} = 0 \qquad K_{22} = (-1)^{2+2}\begin{vmatrix} -2 & -6 \\ 0 & -4 \end{vmatrix} = 8 \qquad K_{32} = (-1)^{3+2}\begin{vmatrix} -2 & -6 \\ 0 & 0 \end{vmatrix} = 0$

$K_{13} = (-1)^{1+3}\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0 \qquad K_{23} = (-1)^{2+3}\begin{vmatrix} -2 & 4 \\ 0 & 0 \end{vmatrix} = 0 \qquad K_{33} = (-1)^{3+3}\begin{vmatrix} -2 & 4 \\ 0 & 1 \end{vmatrix} = -2$

$$K^{-1} = \frac{1}{\det K}\begin{pmatrix} K_{11} & K_{21} & K_{31} \\ K_{12} & K_{22} & K_{32} \\ K_{13} & K_{23} & K_{33} \end{pmatrix} = \frac{1}{8}\begin{pmatrix} -4 & 16 & 6 \\ 0 & 8 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$X = H \cdot K^{-1}$

$$X = \frac{1}{8}\begin{pmatrix} -4 & 4 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -4 & 16 & 6 \\ 0 & 8 & 0 \\ 0 & 0 & -2 \end{pmatrix} = \frac{1}{8}\begin{pmatrix} 16 & -32 & -30 \\ 0 & 16 & 2 \\ 0 & 0 & -2 \end{pmatrix} = \begin{pmatrix} 2 & -4 & -\frac{15}{4} \\ 0 & 2 & \frac{1}{4} \\ 0 & 0 & -\frac{1}{4} \end{pmatrix}$$

3. $\left[(AX)^T - X^T \cdot B\right]^{-1} = A - B$, $A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & 0 \\ 4 & -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

I način

$$E = ((AX)^T - X^T B) \cdot (A - B)$$

$$A^T = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 0 & -1 \\ 1 & 0 & 2 \end{pmatrix}$$

II način

$$(X^T(A-B))^{-1} = A - B$$

III način

$$(AX)^T - X^T B = (A-B)^{-1}$$

$$X^T A^T - X^T B = (A-B)^{-1}$$

$$X^T(A^T - B) = (A-B)^{-1} \quad / (A^T - B)^{-1}$$

$$X^T = \underbrace{(A-B)^{-1}}\underbrace{(A^T-B)^{-1}} \quad /^T$$

$$\cancel{X = ((A^T-B)^{-1})^T((A-B)^{-1})^T}$$

$$A^T - B = \begin{pmatrix} 0 & 3 & 3 \\ 2 & 0 & -2 \\ 0 & -1 & 2 \end{pmatrix}$$

$$X = ((A-B)^{-1} \cdot (A^T - B)^{-1})^T$$

$$A - B = \begin{pmatrix} 0 & 2 & 0 \\ 3 & 0 & -1 \\ 3 & -2 & 2 \end{pmatrix}$$

$$X = (((A^T-B)(A-B))^{-1})^T$$

$$C = (A^T - B)(A - B) = \begin{pmatrix} 18 & -6 & 3 \\ -6 & 8 & -4 \\ 3 & -4 & 5 \end{pmatrix}$$

$$\det C = \begin{vmatrix} 18 & -6 & 3 \\ -6 & 8 & -4 \\ 3 & -4 & 5 \end{vmatrix} = 324$$

Rang matrice

1. Odrediti rang matrice

$$A = \begin{pmatrix} 1 & -2 & 3 & 2 \\ 2 & 1 & -1 & 0 \\ 3 & -1 & 2 & 2 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} -2 & 3 \\ -1 & 2 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 1 & 0 \\ 3 & -1 & 2 \end{pmatrix}$$

$$\det A_1 = \begin{vmatrix} -2 & 3 \\ -1 & 2 \end{vmatrix} = -1 \qquad \det A_2 = \begin{vmatrix} 1 & -2 & 2 \\ 2 & 1 & 0 \\ 3 & -1 & 2 \end{vmatrix} = -2 + 2 = 0$$

$$A = \begin{pmatrix} 1 & -2 & 3 & 2 \\ 2 & 1 & -1 & 0 \\ 3 & -1 & 2 & 2 \end{pmatrix} \cdot(-2), \cdot(-3) \sim \begin{pmatrix} 1 & -2 & 3 & 2 \\ 0 & 5 & -7 & -4 \\ 0 & 5 & -7 & -4 \end{pmatrix} \cdot(-1) \sim \begin{pmatrix} 1 & -2 & 3 & 2 \\ 0 & 5 & -7 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

svaka sl. kolona mora imati više 0

$r(A) = 2$, $M \begin{vmatrix} 1 & -2 \\ 0 & 5 \end{vmatrix} = 5 \neq 0$, a minori reda većeg od 2 su jednaki nuli

2. U zavisnosti od parametra a odrediti rang matrice

$$A = \begin{pmatrix} a & 1 & 0 & -1 \\ 2 & -2 & -1 & -2 \\ -1 & 3 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} -1 & 3 & 1 & 1 \\ 2 & -2 & -1 & -2 \\ a & 1 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 1 & -1 \\ -2 & -2 & -1 & 2 \\ -1 & 1 & 0 & a \end{pmatrix} \cdot 2, + , + \sim$$

$$\sim \begin{pmatrix} 1 & 3 & 1 & -1 \\ 0 & 4 & 1 & 0 \\ 0 & 4 & 1 & a-1 \end{pmatrix} \cdot(-1) \sim \begin{pmatrix} 1 & 3 & 1 & -1 \\ 0 & 4 & 1 & 0 \\ 0 & 0 & 0 & a-1 \end{pmatrix}$$

1) $a \neq 1$

$r(A) = 3$, $M = \begin{vmatrix} 1 & 3 & -1 \\ 0 & 4 & 0 \\ 0 & 0 & a-1 \end{vmatrix} = 4(a-1) \neq 0$

2) $a = 1$

$$A \sim \begin{pmatrix} 1 & 3 & 1 & -1 \\ 0 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$r(A) = 2$, $M = \begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = 4 \neq 0$

Sistemi linearnih j-na

1. Gausovim metodom riješiti sistem

$$\begin{aligned} 2x + y - 2z &= 6 \\ x - 3y + 2z &= -7 \\ -x + 2y - z &= 4 \end{aligned}$$

$$\begin{aligned} x - 3y + 2z &= -7 \quad \big| \cdot(-2) \\ 2x + y - 2z &= 6 \\ -x + 2y - z &= 4 \end{aligned}$$

$$\begin{aligned} x - 3y + 2z &= -7 \\ 7y - 6z &= 20 \\ -y + z &= -3 \end{aligned}$$

$$\begin{aligned} x - 3y + 2z &= -7 \\ -y + z &= -3 \quad \big| \cdot 7 \\ 7y - 6z &= 20 \end{aligned}$$

$$\begin{aligned} x - 3y + 2z &= -7 \\ -y + z &= -3 \\ z &= -1 \end{aligned}$$

$z = -1$

$-y - 1 = -3 \Rightarrow y = 2$

$x = -7 + 3y - 2z \Rightarrow x = 1$

Sistem ima jedinstveno rješenje

$(1, 2, -1)$

2. U zavisnosti od parametra a diskutovati i riješiti sistem primjenom Gausovog algoritma

5 $\quad a-12 \quad \frac{a-12}{5}$

$$\begin{aligned} ax - y + 3z &= a - 1 \\ x + ay - z &= 1 \\ 4x + 3y + z &= 3 \end{aligned}$$

$$\left(\begin{array}{ccc|c} a & -1 & 3 & a-1 \\ 1 & a & -1 & 1 \\ 4 & 3 & 1 & 3 \end{array}\right) \sim \begin{array}{c} \begin{array}{ccc} z & y & x \end{array} \\ \left(\begin{array}{ccc|c} 3 & -1 & a & a-1 \\ -1 & a & 1 & 1 \\ 1 & 3 & 4 & 3 \end{array}\right) \end{array} \sim \left(\begin{array}{ccc|c} 1 & 3 & 4 & 3 \\ -1 & a & 1 & 1 \\ 3 & -1 & a & a-1 \end{array}\right) \cdot(-3) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 3 & 4 & 3 \\ 0 & a+3 & 5 & 4 \\ 0 & -10 & a-12 & a-10 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 4 & 3 \\ 0 & -10 & a-12 & a-10 \\ 0 & a+3 & 5 & 4 \end{array}\right) \cdot \frac{a+3}{10} \sim \left(\begin{array}{cccc} 1 & 3 & 4 & 3 \\ 0 & -10 & a-12 & a-10 \\ 0 & 0 & \frac{(a-2)(a-7)}{10} & \frac{(a-2)(a-5)}{10} \end{array}\right) \cdot 10 \sim$$

$$\sim\left(\begin{array}{ccc|c} 1 & 3 & 4 & 3 \\ 0 & -10 & a-12 & a-10 \\ 0 & 0 & (a-2)(a-7) & (a-2)(a-5) \end{array}\right)$$

$$(a-12)\,\frac{a+3}{10}+5$$

$$\frac{a^2-9a+14}{10}=\frac{(a-2)(a-7)}{10}$$

$$(a-10)\,\frac{a+3}{10}+4$$

$$\frac{a^2-7a+10}{10}=\frac{(a-2)(a-5)}{10}$$

$$z+3y+4x=3$$
$$-10y+(a-12)x=a-10$$
$$(a-2)(a-7)x=(a-2)(a-5)$$

I $\quad a\neq 2,\ a\neq 7$

$(3):\quad x=\dfrac{a-5}{a-7}$

$(2):\quad y=\dfrac{10-a+(a-12)x}{10}=\dfrac{1}{7-a}$

$(1):\quad z=3-3y-4x=\dfrac{2-a}{a-7}$

Sistem je saglasan i određen $\left(\dfrac{a-5}{a-7},\ \dfrac{1}{7-a},\ \dfrac{2-a}{a-7}\right)$

II $\quad a=2$

$$\left(\begin{array}{ccc|c} 1 & 3 & 4 & 3 \\ 0 & -10 & -10 & -8 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$z+3y+4x=3$$
$$-10y-10x=-8$$

x – slobodna nepoznata

$$y=\frac{8-10x}{10}=\frac{4-5x}{5}$$

$$z=3-3y-4x=\frac{3-5x}{5}$$

Sistem je saglasan i neodređen

Opšte rj. je $\left(x,\ \dfrac{4-5x}{5},\ \dfrac{3-5x}{5}\right)$

III $\quad a=7$

$$\left(\begin{array}{cccc} 1 & 3 & 4 & 3 \\ 0 & -10 & -5 & -3 \\ 0 & 0 & 0 & 10 \end{array}\right)$$

$$z+3y+4x=3$$
$$-10y-5x=-3$$
$$0\cdot x=10\ \perp$$

Sistem je nesaglasan

31. oktobar

1. Primjenom Kramerovog pravila riješiti sistem

$$x_1 + 2x_2 + x_3 = 8$$
$$3x_1 + 2x_2 + x_3 = 10$$
$$4x_1 + 3x_2 - 2x_3 = 4$$

$$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 4 & 3 & -2 \end{vmatrix}\begin{matrix} 1 & 2 \\ 3 & 2 \\ 4 & 3 \end{matrix} = -4 + \cancel{8} + 9 - \cancel{8} - 3 + 12 = 14 \neq 0 \Rightarrow$$ Sistem je saglasan i određen

$$D_1 = \begin{vmatrix} 8 & 2 & 1 \\ 10 & 2 & 1 \\ 4 & 3 & -2 \end{vmatrix} = \ldots = 14 \qquad x_1 = \frac{D_1}{D} = \frac{14}{14} = 1$$

$$D_2 = \begin{vmatrix} 1 & 8 & 1 \\ 3 & 10 & 1 \\ 4 & 4 & -2 \end{vmatrix} = \ldots = 28 \qquad x_2 = \frac{D_2}{D} = \frac{28}{14} = 2$$

$$D_3 = \begin{vmatrix} 1 & 2 & 8 \\ 3 & 2 & 10 \\ 4 & 3 & 4 \end{vmatrix} = \ldots = 42 \qquad x_3 = \frac{D_3}{D} = \frac{42}{14} = 3$$

Rješenje sistema je $(1, 2, 3)$

2. Koristeći Kramerovo pravilo riješiti sistem

$$ax - y + 3z = a - 1$$
$$x + ay - z = -1$$
$$4x + 3y + z = 3$$

$$D = \begin{vmatrix} a & -1 & 3 \\ 1 & a & -1 \\ 4 & 3 & 1 \end{vmatrix} = (a-7)(a-2)$$

$$D_1 = \begin{vmatrix} a-1 & -1 & 3 \\ -1 & a & -1 \\ 3 & 3 & 1 \end{vmatrix} = (a-2)(a-5)$$

$$D_2 = \begin{vmatrix} a & a-1 & 3 \\ 1 & -1 & -1 \\ 4 & 3 & 1 \end{vmatrix} = -(a-2)$$

$$D_3 = \begin{vmatrix} a & -1 & a-1 \\ 1 & a & -1 \\ 4 & 3 & 3 \end{vmatrix} = -(a-2)^2$$

Kritične vrijednosti $a=7$ i $a=2$

1) $a \neq 7$, $a \neq 2 \Rightarrow D \neq 0 \Rightarrow$ Sistem je saglasan i određen

$$x = \frac{D_1}{D} = \frac{\cancel{(a-2)}(a-5)}{(a-7)\cancel{(a-2)}} = \frac{a-5}{a-7}$$

$$y = \frac{D_2}{D} = \frac{-\cancel{(a-2)}}{(a-7)\cancel{(a-2)}} = -\frac{1}{a-7}$$

$$z = \frac{D_3}{D} = \frac{-(a-2)^2}{(a-7)\cancel{(a-2)}} = \frac{-(a-2)}{a-7}$$

Rj. sistema je $\left(\frac{a-5}{a-7}, \frac{-1}{a-7}, \frac{-(a-2)}{a-7}\right)$

2) $a=7 \Rightarrow D=0$

$D_1 = (7-2)(7-5) = 5\cdot 2 = 10 \neq 0 \Rightarrow$ Sistem je nesaglasan

3) $a=2 \Rightarrow D=0=D_1=D_2=D_3$

$$\begin{aligned} 2x - y + 3z &= 1 \\ x + 2y - z &= -1 \\ 4x + 3y + z &= 3 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 1 & 2 & -1 & -1 \\ 4 & 3 & 1 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 2 & -1 & 3 & 1 \\ 4 & 3 & 1 & 3 \end{array}\right)^{(-2)\,(-4)} \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & -5 & 5 & 3 \\ 0 & -5 & 5 & 7 \end{array}\right)^{(-1)} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & -5 & 5 & 3 \\ 0 & 0 & 0 & 4 \end{array}\right)$$

$$\begin{aligned} x + 2y - z &= -1 \\ -5y + 5z &= 3 \\ 0 &= 4 \end{aligned}$$

$\Rightarrow$ Sistem je nesaglasan

3. Koristeći Kramerovo pravilo riješiti sistem.

$$\begin{aligned} ax + y + z &= 1 \\ x + ay + z &= 2 \\ x + y + az &= -3 \end{aligned}$$

$$D = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = - \begin{vmatrix} 1 & 1 & a \\ 1 & a & 1 \\ a & 1 & 1 \end{vmatrix} = - \begin{vmatrix} 1 & 1 & a \\ 0 & a-1 & 1-a \\ 0 & 1-a & 1-a^2 \end{vmatrix} = - \begin{vmatrix} 1 & 1 & a \\ 0 & a-1 & 1-a \\ 0 & 1-a & (1-a)(1+a) \end{vmatrix} =$$

$$= - \begin{vmatrix} 1 & 1 & a \\ 0 & a-1 & 1-a \\ 0 & 0 & (1-a)(2+a) \end{vmatrix} = -(a-1)(1-a)(2+a) = (1-a)^2(2+a)$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & a & 1 \\ -3 & 1 & a \end{vmatrix} = (a-1)(a+2)$$

$$D_2 = \begin{vmatrix} a & 1 & 1 \\ 1 & 2 & 1 \\ 1 & -3 & a \end{vmatrix} = 2(a-1)(a+2)$$

$$D_3 = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 2 \\ 1 & 1 & -3 \end{vmatrix} = -3(a-1)(a+2)$$

Kritične vr. $a = 1 \wedge a = -2$

1) $a \neq 1 \wedge a \neq -2 \Rightarrow D \neq 0 \Rightarrow$ Sistem je saglasan i određen

$$x = \frac{D_1}{D} = \frac{\cancel{(a-1)}\cancel{(a+2)}}{(a-1)^{\cancel{2}}\cancel{(a+2)}} = \frac{1}{a-1}$$

$$y = \frac{D_2}{D} = \frac{2\cancel{(a-1)}\cancel{(a+2)}}{(a-1)^{\cancel{2}}\cancel{(a+2)}} = \frac{2}{a-1}$$

$$z = \frac{D_3}{D} = \frac{-3\cancel{(a-1)}\cancel{(a+2)}}{(a-1)^{2}\cancel{(a+2)}} = \frac{-3}{a-1}$$

Rj. sistema je $\left(\frac{1}{a-1}, \frac{2}{a-1}, \frac{-3}{a-1} \right)$

2) $a = 1 \Rightarrow D = 0 = D_1 = D_2 = D_3$

$$\left.\begin{aligned} x + y + z &= 1 \\ x + y + z &= 2 \\ x + y + z &= -3 \end{aligned}\right\} \Rightarrow \text{Sistem je nesaglasan}$$

3) $a = -2 \Rightarrow D = 0 = D_1 = D_2 = D_3$

$$\begin{aligned} -2x + y + z &= 1 \\ x - 2y + z &= 2 \\ x + y - 2z &= -3 \end{aligned}$$

$$\left(\begin{array}{ccc|c} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 2 \\ 1 & 1 & -2 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -2 & -3 \\ 1 & -2 & 1 & 2 \\ -2 & 1 & 1 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & -2 & -3 \\ 0 & -3 & 3 & 5 \\ 0 & 3 & -3 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -2 & -3 \\ 0 & -3 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{aligned} x + y - 2z &= -3 \\ -3y + 3z &= 5 \end{aligned}$$

$\Rightarrow$ Sistem je saglasan i neodređen

Opšte rješenje je $\left(\frac{-4 + 3z}{3}, \frac{3z - 5}{3}, z \right)$ $z \in \mathbb{R}$

4. Primjenom K - K teoreme diskutovati i riješiti sistem u zavisnosti od parametra m.

$$\begin{aligned} mx + y + z &= 1 \\ x + (3-m)y + z &= m \\ mx + (m+1)y + z &= 1 - m \end{aligned}$$

$$(A|b) = \left(\begin{array}{ccc|c} m & 1 & 1 & 1 \\ 1 & 3-m & 1 & m \\ m & m+1 & 1 & 1-m \end{array}\right) \sim \begin{array}{c} \begin{array}{ccc} z & y & x \end{array} \\ \left(\begin{array}{ccc|c} 1 & 1 & m & 1 \\ 1 & 3-m & 1 & m \\ 1 & m+1 & m & 1-m \end{array}\right) \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & m & 1 \\ 0 & 2-m & 1-m & m-1 \\ 0 & m & 0 & -m \end{array}\right) \sim \begin{array}{c} \begin{array}{ccc} z & x & y \end{array} \\ \left(\begin{array}{ccc|c} 1 & m & 1 & 1 \\ 0 & 1-m & 2-m & m-1 \\ 0 & 0 & m & -m \end{array}\right) \end{array}$$

mogu da smanje rang matrice

Kritične vr. su $m=1$ i $m=0$

1) $m \neq 0$ i $m \neq 1$

$\operatorname{rang} A = 3$ jer postoji minor $\begin{vmatrix} 1 & m & 1 \\ 0 & 1-m & 2-m \\ 0 & 0 & m \end{vmatrix} = (1-m)m \neq 0$, a minora većeg reda nema

$\operatorname{rang}(A|b) = 3$ — || —

$n = 3$

$\operatorname{rang} A = \operatorname{rang}(A|b) = n \xRightarrow{\text{po k-k t.}}$ Sistem je saglasan i određen

$z + mx + y = 1$

$(1-m)x + (2-m)y = m-1$

$\underline{my = -m} \Rightarrow y = -1$

$z + mx - 1 = 1$

$\underline{(1-m)x - (2-m) = m-1}$

$z + mx = 2$

$\underline{(1-m)x = 1} \Rightarrow x = \frac{1}{1-m}$

$z + \frac{m}{1-m} = 2$

$z = 2 - \frac{m}{1-m} \Rightarrow z = \frac{2-3m}{1-m}$

Rješenje sistema je $\left(\frac{1}{1-m}, -1, \frac{2-3m}{1-m} \right)$

2) $m = 0$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$\operatorname{rang} A = 2$ jer postoji minor $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$ a svi minori većeg reda su 0

$\operatorname{rang}(A|b) = 2$

$n = 3$

$\operatorname{rang} A = \operatorname{rang}(A|b) < n \xRightarrow{\text{po k-k t.}}$ Sistem je saglasan i neodređen

$z + y = 1 \Rightarrow z = 1 - y$

$x + 2y = -1 \Rightarrow x = -1 - 2y$

Opšte rj. je $(-1-2y,\ y,\ 1-y)$ $y \in \mathbb{R}$.

3) $m = 1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right) (-1) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

rang $A = 2$ jer postoji minor $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1 \neq 0$, a svi minori većeg reda su 0

rang $(A|b) = 3$ jer postoji minor $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -1 \neq 0$, a minora većeg reda nema

rang $A <$ rang $B \xRightarrow{\text{po K-K t.}}$ Sistem je nesaglasan

5. U zavisnosti od parametra a diskutovati i riješiti sistem

$$\begin{aligned} x + y + z &= 0 \\ ax + 4y + z &= 0 \\ 6x + (a+2)y + 2z &= 0 \end{aligned}$$

$$\begin{array}{ccc} x & y & z \\ \end{array} \qquad \begin{array}{ccc} z & y & x \end{array}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ a & 4 & 1 \\ 6 & a+2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 4 & a \\ 2 & a+2 & 6 \end{pmatrix} (-1)\ (-2) \sim$$

$$\begin{array}{ccc} z & y & x \end{array}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & a-1 \\ 0 & a & 4 \end{pmatrix} \text{II}\cdot a + \text{III}\cdot 3 \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & a-1 \\ 0 & 0 & -a^2+a+12 \end{pmatrix}$$

$-a^2 + a + 12 = 0$

$a_{1,2} = \dfrac{-1 \pm 7}{-2}$

$a_1 = 4$, $a_2 = -3$

Kritične vrijednosti $a = 4$ i $a = -3$

1) $a \neq 4 \wedge a \neq -3$

rang $A = 3$, jer postoji minor $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 3 & a-1 \\ 0 & 0 & -a^2+a+12 \end{vmatrix} = 3(-a^2 + a + 12) \neq 0$

$n = 3$

rang $A = n \xRightarrow{\text{po K-K t.}}$ Sistem je saglasan i određen

$$z + y + x = 0$$
$$3y + (a-1)x = 0$$
$$\underline{(-a^2+a+12)x = 0}$$
$$x = y = z = 0$$

Rješenje je $(0,0,0)$

2) $a = 4$

$$\begin{array}{ccc} z & y & x \\ \end{array}$$
$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$z + y + x = 0$$
$$\underline{3y + 3x = 0} \Rightarrow y = -x$$
$$z = 0$$

Opšte rješenje je

$(x, -x, 0)$ $x \in \mathbb{R}$

3) $a = -3$

$$\begin{array}{ccc} z & y & x \\ \end{array}$$
$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & -4 \\ 0 & 0 & 0 \end{pmatrix}$$

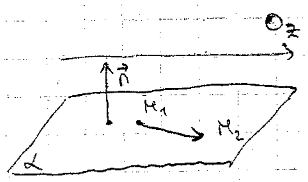
$$z + y + x = 0$$
$$\underline{3y - 4x = 0} \Rightarrow y = \frac{4}{3}x$$
$$z = -\frac{7}{3}x$$

Opšte rj. je

$(x, \frac{4}{3}x, -\frac{7}{3}x)$ $x \in \mathbb{R}$

Prava i ravan

1. Naći j-nu ravni koja je paralelna sa osom Oz, a sadrži tačke $M_1(2,2,0)$ i $M_2(4,0,0)$



$$\overrightarrow{M_1M_2} = (2, -2, 0)$$

$$\vec{k} \times \overrightarrow{M_1M_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 2 & -2 & 0 \end{vmatrix} = 2\vec{i} - \vec{j}(-2) + \vec{k}\cdot 0 =$$
$$= (2, 2, 0)$$

$$\vec{k} = (0,0,1)$$

$$\left.\begin{array}{l} \vec{n} \perp \vec{k} \\ \vec{n} \perp \overrightarrow{M_1M_2} \end{array}\right\} \Rightarrow \vec{n} = \lambda(\vec{k} \times \overrightarrow{M_1M_2})$$

$$\vec{n} = \lambda(2,2,0)$$
$$\lambda = \frac{1}{2}$$

$$\left.\begin{array}{l} \vec{n} = (1,1,0) \\ M_2 = (4,0,0) \end{array}\right\} \begin{array}{l} 1(x-4) + 1(y-0) + 0(z-0) = 0 \\ \boxed{\alpha: x + y - 4 = 0} \end{array}$$